RHEOLOGICAL PROPERTIES OF SOME CLAY SOILS

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An investigation is made of the deformation properties of clays in uniaxial and triaxial compression at constant moisture content. Rheological equations of state for clay soils are proposed for the three-dimensional case.

Deformation of clay soils under prolonged loading may be regarded as the sum of three deformations of different types – instantaneous elastic, elastic aftereffect, and viscous flow:

$$\varepsilon_x = \varepsilon_{xi} + \varepsilon_{xa} + \varepsilon_{xf}, \quad (x, y, z). \tag{1}$$

All three components retain their fundamental properties during arbitrary deformation time and can be separated at any time by removing the load. The superposition principle is therefore valid for strains in clay soils, just as in certain other media [1].

In these investigations we used several varieties of marly fossiliferous and black Jurassic clays of semi-hard and hard consistency and a degree of water saturation close to unity. All tests were performed under conditions of uniaxial and triaxial compression on cylindrical specimens with a diameter-to-height ratio of 1: 2. 5. The specimens were sealed to avoid the possibility of a change of moisture content due to drying out or draining.

The volume change for clay soils under hydrostatic pressure is almost completely reversible, there being no appreciable creep strain (elastic aftereffect or viscous flow). When the specimens are subjected to a complete loading cycle under hydrostatic pressure, elastic hysteresis is invariable observed. The axial points of the hysteresis loop are located along a straight line (Fig. 1), i. e., the volume change strains are proportional to the hydrostatic stresses:

$$\varepsilon_m = \sigma_m / K. \tag{2}$$

It follows from the tests on clay samples in hydrostatic compression that the volume change due to mean normal (hydrostatic) stress is associated only with the instantaneous elastic strain, whereas creep strains are caused only by stress deviator components and are not accompanied by volume change. This is confirmed by direct measurements of the transverse creep strain [2, 3] – in this case the values of Poisson's ratio are very close to the limiting value of 0. 5. Since there are no volume creep strains, it is clear that the experimentally measured strains due to elastic aftereffect and viscous flow are equal to the corresponding components of the deviators for these strains.

The tests show that the instantaneous elastic strains are uniquely determined by the applied stresses, and do not depend on their duration of action. The stress dependence of the instantaneous elastic strains may be obtained by progressively loading the specimens, if, after applying each successive load step, the whole load is rapidly removed and the corresponding elastic recovery is determined. For the uniaxial stress state this nonlinear relation is well described by the function

$$\varepsilon_{xi} = \sigma_x / E_i(\sigma_x) = \sigma_x / E_0 (1 - b\sigma_x),$$

and for triaxial stress, by the analogous function

$$(\varepsilon_x - \varepsilon_m)_i = \frac{\sigma_x - \sigma_m}{2G_i(\sigma_j)} = \frac{\sigma_x - \sigma_m}{2G_0(1 - a\sigma_j)}. \tag{3}$$

To calculate ε_m the specimen must first be subjected to hydrostatic compression.

If values of the variable moduli $E_i(\sigma_j) = \sigma_x/\varepsilon_{x\,i}$ and $G_i(\sigma_j) = (\sigma_x - \sigma_m)/(\varepsilon_x - \varepsilon_m)_i$ and the constant modulus K are determined from the test data, then values of Poisson's ratio for instantaneous elastic strains may also be found:

$$\mu_i = E_i(\sigma_j)/2G_i(\sigma_j) - 1, \quad \text{or} \quad \mu_i = 1/2 - E_i(\sigma_j)/2K.$$

It follows from the last formula that Poisson's ratio should be a linear function of the stress intensity:

$$\mu_M(\sigma_i) = \mu_0(1 + k\sigma_i),$$

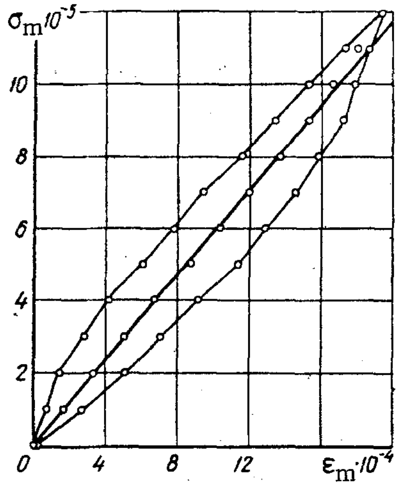

Fig. 1. Volume change under hydrostatic compression for a sample of Jurassic clay.

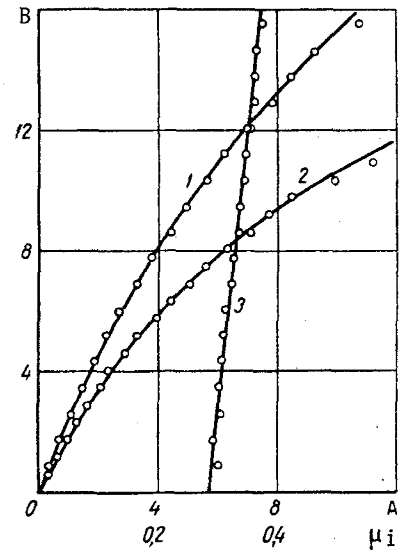

Fig. 2. Instantaneous elastic strain of a sample of Jurassic clay: 1) in uniaxial compression; 2) as a function of $\sigma_x - \sigma_m$; 3) change in Poisson's ratio.

where $\mu_0 = (K - E_0)/2K$ and $k = b\,E_0/(K - E_0)$. At low stresses this ratio lies within the range of mean values 0.25-0.30, and increases (Fig. 2) to 0.35-0.40 at stresses close to failure. Determination of Poisson's ratio from measurements of the instantaneous transverse strain leads to similar results.

In the initial stage directly after loading or unloading the strains due to elastic aftereffect are comparatively rapid, but the strain rate is very quickly reduced as the strains as asymptotically approach their limiting values, which depend on the applied stresses. Only the reverse elastic aftereffect can be determined directly from the test data, since the ini-

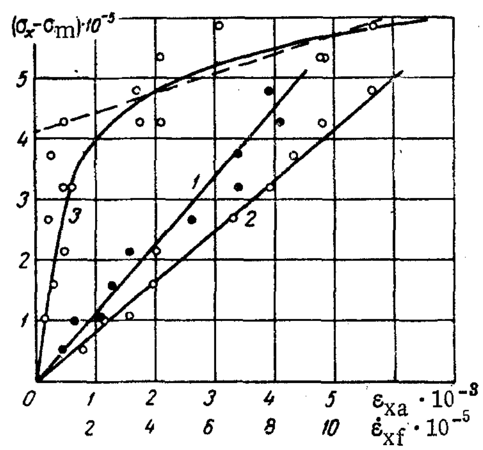

Fig. 3. Long-time strains in samples of Jurassic clay: 1) elastic aftereffect at t = 24 hr; 2) the same at t = 29 days; 3) viscous flow strain rate.

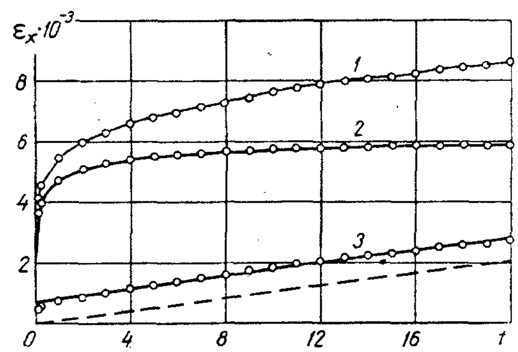

Fig. 4. Time-dependence of long-time strains in a sample of Jurassic clay for $\sigma_x - \sigma_m = 5.3 \cdot 10^5\ N/m^2$: 1) creep strain; 2) elastic aftereffect; 3) viscous flow.

tial elastic aftereffect is accompanied by irreversible viscous flow. To study the elastic aftereffect, a series of samples must be tested at different, but constant loads. If the load is applied long enough, a condition is reached where the strain grows at a certain steady rate. This indicates that the initial elastic aftereffect has almost attained its limiting value, and that subsequent increase in creep strain will be due mainly to viscous flow of the material. Following rapid removal of the load the development of the reverse elastic aftereffect may be traced.

The results of the investigations lead to the conclusion that at any instant the elastic aftereffect in clay soils depends linearly on the stress deviator components (Fig. 3.). The Kohlrausch-Bronskii function [4] gives a very good approximation of the growth of these strains (Fig. 4.):

$$\varepsilon_{xa} = \frac{\sigma_x - \sigma_m}{2G_a}\,[1 - \exp(-\beta t^{1-\alpha})]. \tag{4}$$

The dimensionless parameter α is a kind of universal constant, its value being almost the same for widely different materials. Its mean values for clay soils lie in the very narrow range 0.73-0.77, whereas for rubber $\alpha = 0.75$ [4], and for metals $\alpha \approx 0.70$ [5].

Viscous flow strains may be determined as the difference between the creep strain and reverse elastic aftereffect at the corresponding times. At constant stress, the viscous flow strain tends to increase without limit at some steady rate (Fig. 4.), and if we neglect the small initial jump, we may assume that these strains are proportional to duration of application of the load:

$$\varepsilon_{xf} = \dot{\varepsilon}_{xf} t. \tag{5}$$

Viscous flow may be observed even at very low stresses, although its rate is then very small (Fig. 3.). The behavior of the soil changes considerably, however, as the stress grows, and the viscous flow strain rate rapidly increases. The following equation of state of a viscous liquid with variable viscosity is in quite satisfactory agreement with the experimental data

$$\dot{\varepsilon}_{xf} = (\sigma_x - \sigma_m)/2\eta = (\sigma_x - \sigma_m)/2(\zeta_0 + c\,\sigma_j^m)^{-1}, \tag{6}$$

here the exponent m is an even integer characterizing the rate of decrease of the viscosity of the material with increase in stress. For Jurassic and marly clays the most suitable value is m = 6.

It may sometimes be more convenient to use an approximation – the equation of a viscoplastic body given in [6] (broken line in Fig. 3):

$$\text{when } \sigma_i < \sigma_T \quad \dot{\varepsilon}_{xf} = 0,$$

$$\text{when } \sigma_i > \sigma_f \quad \dot{\varepsilon}_{xf} = \frac{1}{2}\zeta\left(1 - \frac{\sigma_T}{\sigma_i}\right)(\sigma_x - \sigma_m).$$

Clay soils are complex media, possessing in some degree the properties of a nonlinear viscous liquid. To describe the deformation of clay soils we may use the equations of the Boltzmann-Volterra memory theory, generalized for nonlinear media by Rabotnov and Rozovskii [1, 7]. These general equations must be transformed, using (1), (2) and (3) and the strain rate from (4) and (6). Then

$$\begin{aligned}
\varepsilon_x &= \frac{\sigma_m}{K} + \frac{\sigma_x - \sigma_m}{2G_0(1 - a\sigma_j)} + \\
&+ \frac{\beta(1-\alpha)}{2G_a}\int_0^t (t-\xi)^{-\alpha}\exp[-\beta(t-\xi)^{1-\alpha}][\sigma_x(\xi) - \sigma_m(\xi)]\,d\xi + \\
&+ \frac{1}{2}\int_0^t [\zeta_0 + c\,\sigma_j^m(\xi)][\sigma_x(\xi) - \sigma_m(\xi)]\,d\xi, \\
\gamma_{xy} &= \frac{\tau_{xy}}{G_0(1 - a\sigma_j)} + \frac{\beta(1-\alpha)}{G_a}\int_0^t (t-\xi)^{-\alpha}\exp[-\beta(t-\xi)^{1-\alpha}]\,\tau_{xy}(\xi)\,d\xi + \\
&+ \int_0^t [\zeta_0 + c\,\sigma_j^m(\xi)]\,\tau_{xy}(\xi)\,d\xi \qquad (x,\ y,\ z).
\end{aligned} \tag{7}$$

Equations (7) should be used for active deformation processes. The same equations will also be suitable for the case of unloading, if in each of them we neglect the last term on the right, corresponding to irreversible viscous flow.

NOTATION

$\sigma_x, \ldots, \tau_{xy}, \ldots$ – normal and tangential stresses; $\varepsilon_x, \ldots, \gamma_{xy}, \ldots$ – relative linear strains and shear strains; σ_m and ε_m – mean stress and strain; σ_j – stress intensity; σ_f – yield point: K – bulk modulus; E_0 – initial elastic modulus;

G_0 – initial, and G_a – long-time shear moduli; a and b – instantaneous elastic strain parameters; α and β – elastic aftereffect parameters; ζ_0, ζ, c and m – viscous flow parameters; t – time; ξ – variable of integration. Stresses and strains are treated as functions of the coordinates (x, y, z) and time.

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